# Rate Selection Heuristics for Network Coding in Wireless Networks

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## 1. OVERVIEW

We address the problem of efficient broadcasting in multi-hop wireless networks with network coding. Network coding was introduced by the seminal work of [1] as a new paradigm where intermediate nodes are mixing information from different flows (different bits or different packets). The problem that we are addressing is *efficient broadcast*, precisely:

- Broadcast packets from one source to all nodes, with the minimum number of transmissions.

Without network coding, finding the optimal solution is an NP complete problem. With network coding, *essentially*, nodes will retransmit coded packets with an average interval, defining a node rate. Finding an optimal solution consists in finding the coding nodes and their optimal rates. This can be formulated as a linear program, which can be solved in polynomial time [2].

However, we adopt a different, even simpler, approach: our previous work [3, 4] has shown a simple heuristic could achieve asymptotically the optimal efficiency for homogeneous large and dense wireless networks of the plane — and also that, noticeably, *it outperforms methods not using network coding.* This is true asymptotically, and for homogeneous networks. The heuristic need adjustments for less homogeneous, smaller or sparser networks, the topic of the poster. Our key contributions are the following:

- We propose an improved heuristic for rate selection, inspired by [5]. It is based only on local topology information: knowledge of two-hop neighbors.
- We study its performance on representative graphs with different densities (using min-cut calculation). We investigate and explain the variation of the performance.

## 2. DEFINITIONS

### 2.1 Network Model

We assume an ideal wireless model, infinite capacity: lossless wireless transmissions without collisions or interferences. Wireless networks are usually modelled as *unit disk graphs* of the plane, where two nodes are neighbors whenever their distance is lower than a fixed radio range ; see Fig. 1 the principle of unit disk graphs. In addition, in wireless networks, the *wireless broadcast advantage* is used: each transmission is overheard by several nodes. As a result the graph is in reality a *(unit disk) hypergraph.* Precisely, we consider the following networks: unit disk [hyper]graphs where nodes are either distributed randomly (Fig. 1(a)) or more regularly organized in a lattice (Fig. 1(b)). In addition, in both cases, we also consider their variants where the network is a torus, with wrap-around connections in both the $x$ and $y$ directions as on Fig. 1(c).

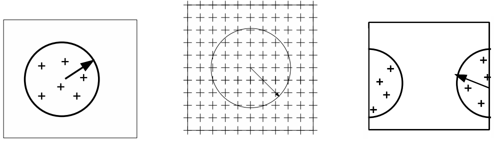
(a) random unit disk graph (b) lattice graph (c) on a torus

Figure 1: Network Models

### 2.2 Performance

Let $C_v$ denote the retransmission rate of a node $v$. Recall that a network coding solution consists of the definition the rate of each node.The metric for the cost is the number of transmissions per broadcast. Consider:

- the number of retransmissions from every node per unit time (directly given by selected rate).
- the number of packets successfully broadcast from the source to the entire network per unit time; it is the achievable broadcast rate.

The ratio between the two is our metric for the cost per broadcast, and is denoted $E_{\text{cost}}$. For reference, we will also use the cost of the optimal solution, $E_{\text{optimal}}$, obtained by solving the linear program presented in [2].

Here, for heuristics, the missing part is the maximum achievable broadcast rate. A central result for network coding in wireless networks, is that it can be computed as the *min-cut* from the source to every destination, with the network considered as a hypergraph [6].

## 3. THE HEURISTIC: IR-MS

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Our starting point is the results from [3, 4]: we proposed a simple heuristic, where most nodes had the same rate, except the source and some nodes near the edge of the network. Then it was shown that asymptotically, for unit disk graphs, near-optimal efficiency was achieved, where every transmission would bring *innovative* information to almost every receiver.

In order to extend it, let us reproduce the logic of [3]: assume a network where every node has rate 1, e.g. one packet per second. Then every node, with $M$ neighbors, can receive $M$ packets per second. Hence the source should inject at least $M$ packets per second. With these rates, then, it turned out that actually the achievable broadcast capacity (the min-cut) is asymptotically $M$.

This is valid asymptotically for large homogeneous dense networks, but need to be adapted, for sparse networks: first of all, some nodes in sparse networks may have significantly less than $M$ neighbors. In that case, they receive less than $M$ packets per second, and obviously are bottlenecks if the targeted broadcast rate is $M$. We name them *starving nodes*. To alleviate the bottleneck at the starving nodes, their neighbors compensate them by increasing their rates with the following heuristic, inspired by [5]:

- IR-MS (Increased Rate for Most Starving node): the rate of a node $v$ is set to $C_v$, with: $C_v = \frac{M}{min_{u \in H_v}(|H_u|)}$, where $H_w$ is the set of neighbors of $w$, and $M$ is the source rate.

## 4. SIMULATION RESULTS

We evaluate the efficiency of the heuristic, as the relative cost w.r.t. the optimal: $E_{\text{rel}-\text{eff}} = \frac{E_{\text{optimal}}}{E_{\text{cost}}} \leq 1$.

Two reference points are the following: first, the asymptotically near-optimal rate selection from [4] will achieve asymptotically an efficiency of 1, but at best 0.2 on our generated small graphs without torus effect. The second reference point is the approximative upper bound in [4], about the achievable performance without network coding, which translates into: $E^{(\text{no}-\text{coding})}_{\text{bound}-\text{rel}-\text{eff}} \approx 0.609\ldots$.

For comparison purposes, we evaluated the IR-MS heuristic on instances of lattice unit disk graphs and random unit disk graphs, both with and without torus effect. Our parameters are the following: number of nodes $N = 196$, the avg. number of neighbors is successively $4, 12, 28, 48, 80$. Fig. 2 represents the efficiency obtained for different cases (average of 10 results). The central result is the appreciable performance of IR-MS compared to our previous reference points: it mostly outperforms the upper bound without network coding (0.609), and always the efficiency of [4] $< 0.2$.

Now consider the four different types of networks:

• First type, the most regular graphs: lattice unit disk graphs on torus. IR-MS achieves almost perfect optimality. This is because rate adjustment is not needed.

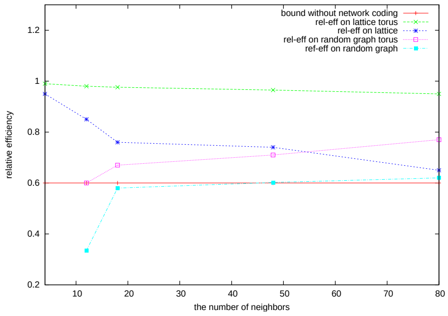

**Figure 2: Relative cost (efficiency)**

• Second type, lattice unit disk graphs without torus: here, the nodes near the border are the issue. IR-MS successful resolves the border issue, as exemplified by its achieving the targeted maximum broadcast rate, $\frac{\text{min}-\text{cut}}{M} = 1$ (see poster [7]). However efficiency decreases as density increases, because of an increase of the cost of nodes near the border.

• Third type, random unit disk graph on torus. No border effect here, but IR-MS has to overcome the effects of non-homogeneity, which is done convincingly.

• Fourth type, genuine random unit disk graph. The performance is acceptable, but on low density, performance is lower, because IR-MS does not fully achieve the maximum broadcast rate (see poster [7]).

## 5. CONCLUSION

We studied a heuristic for efficient broadcasting with network coding only using static local information: one hop or two hop neighbors. We have show excellent performance of this rate selection, and detailed reasons for variations of performance.Future work includes the use of dynamic network coding information in complement of the static local topology information.

## 6. REFERENCES

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